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DEFENSE COMMUNICATIONS ENGINEERING CENTER

TECHNICAL NOTE NO. 15-79

DELAY ANALYSIS OF TASI WITH RANDOM FLUCTUATIONS IN THE NUMBER OF VOICE CALLS

SEPTEMBER 1979

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In this technical note we consider the performance of a voice communications system where the talkspurts of a conversation are buffered and multiplexed over the transmission channels. A mathematical model is given for the behavior of a talkspurt in the system when the voice calls are allowed to statistically come and go. A numerical analysis is conducted to determine the sensitivity of various system measures of performance to the presence of voice calls and to relate various measures of performance.

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SEPTEMBER 1979

Prepared by:

- M. Fischer

Approved for Publication:

JAMES R. RAMOS, LTC, USA
Acting Chief, Systems Engineering Division

FOREWORD

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Director
Defense Communications Engineering Center
1860 Wiehle Avenue
Reston, Virginia 22090

EXECUTIVE SUMMARY

In this technical note we describe a mathematical model used to analyze the delay of a voice talkspurt on a link where talkspurts are buffered when a channel is not available for transmission. The random fluctuation in the number of voice calls are also considered in the model. Numerical application of this model shows:

a. The sensitivity of average buffer delay to the random fluctuations in the number of voice calls.

b. The 95th percentile point of the buffer delay distribution was approximately seven times the average delay for the cases considered in this technical note.

c. The percentage of talkspurts that see a greater than average delay for the smaller number of voice channels might not be acceptable in practice.

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I. INTRODUCTION

The concept of TASI (Time Assignment Speech Interpolation) was first experimented with in the late 1940's and early 1950's by AT&T. In 1958 and 1961 two papers [1], [2] were written concerning the engineering aspects of TASI as well as documenting results of the first actual implementation of TASI on the transatlantic submarine cable channels. The basic idea behind TASI is that in a normal telephone conversation there are gaps and pauses between words and syllables, which allows the active parts of two or more conversations to be multiplexed over the same transmission channel and hence reduce the total number of required transmission channels. The active part of a conversation was called a talkspurt, and since there was no way to buffer the contents of the talkspurt, many talkers and channels were needed in order to take full advantage of the TASI concept.

With the introduction of packet-switched communications systems, interest has developed in the TASI implication of packetized speech. Lincoln Labs [3] considered the network implication of packetized speech, i.e., the effects packetized speech has on overall voice quality, acceptability, and communicability when the voice conversations are transmitted in a packet-switched network. Packetizing the active parts of a voice conversation (the talkspurts) in effect is a switching concept where the philosophy of TASI is implemented in a digital network, and the packets generated by a talkspurt may be buffered when a channel is not available. Weinstein and Hofstetter conducted a study [4] that considered the tradeoff between packet delay and

the TASI advantage in the environment. They showed that the packetized system offers substantial improvements in the TASI advantage (number of talkers/number of channels) even when the number of talkers is small, if one allows the voice packets to experience some delay.

The basic analysis tool used in reference [4] was a simulation model; later a mathematical model was developed for the same system [5]. In all of the analyses performed to date on TASI, the number of talkers was always held fixed and not allowed to fluctuate. The reason was that the statistical fluctuations in the presence of these talkers were much slower than the statistical fluctuations in the generation and transmission of the voice packets. Of course, in reality these talkers were coming and going via some random process and, therefore, the results obtained by fixing the number of talkers could be significantly different than when the talkers were allowed to come and go. In this technical note, we develop a mathematical model that considers these statistical fluctuations (section II) and among other things make comparisons with the case where the number of calls were held fixed (section III). Section IV contains a few remarks and conclusions.

II. MATHEMATICAL ANALYSIS

In this section we give a mathematical analysis of a system when the talkspurts from several voice conversations are multiplexed over C channels. The talkspurts are buffered when there is no free channel, and so in essence, we are considering a packet-switched system. The maximum number of voice conversations that are allowed in the system is N with N/C($N \geq C$) being known as the TASI advantage [4]. In steady state, let Q be the random variable representing the number of voice calls present and Q_T the random variable representing the number of talkspurts that are in the system, i.e., on the channel or in the buffer. If one defines $P_{i,j} = \Pr\{Q=i, Q_T=j\}$ for $i=0,1,\ldots,N$ and $j=0,1,\ldots,i$, then we give a system of equations that $P_{i,j}$ must obey. A solution is given and then used to determine the average number of talkspurts in the system, the average delay a talkspurt encounters in the buffer and the complete delay distribution of a talkspurt in the buffer. Similar results are also obtained for a system where the talkspurt is not allowed to wait in the buffer longer than a specified time. For that system we also give an expression for the probability a talkspurt is disregarded or frozen out. The following assumptions are made:

(1) Voice calls arrive via a Poisson process with parameter λ. The holding time of the call is exponentially distributed with mean μ^{-1}.

(2) An arriving call finding N calls in the system leaves without receiving service. If accepted the arriving call immediately generates a talkspurt. Calls may end at any time; i. e., they do not have to be in a talkspurt, but if they end in a talkspurt the talkspurt also ends.

(3) When there are i ($i \geq 1$) voice calls in the system each call can be either in a silent period or in the process of generating a talkspurt and waiting for it to be transmitted. The length of a silent period has an exponential distribution with mean α^{-1}. The length of time to transmit a talkspurt is exponentially distributed with mean β^{-1}. The talkspurts are served by the C-channels; an arriving talkspurt finding all the channels occupied is buffered. Thus, if a conversation has just generated a talkspurt that has not been transmitted then other talkspurts from that conversation are not allowed to be generated; the number of talkspurts in the system cannot exceed the number of calls present.

From the first two assumptions, one can see that the behavior of the random variable, Q, the number of voice calls present in the system can be modeled via the results of an M/M/N/N loss system (see Cooper [6]). That is, if $\rho=\lambda/\mu$, then

$$\Pr\{Q=i\} = \frac{\rho^i/_{i!}}{\sum_{r=0}^{N} \rho^r/_{r!}} \quad : \quad i=0,1,2,\ldots,N \tag{1}$$

with $\Pr\{Q=N\}= E_B(\rho,N)$ being Erlang's Loss Formula. Equation (1) gives the probability an arriving call is blocked.

In general, the balance equations for $P_{i,j}$ can be written as for $i \leq C-1$:

$$\lambda P_{0,0} = \mu P_{1,0}+\mu P_{1,1}$$

$$(\lambda+i\mu+i\alpha)P_{i,0} = (i+1)\mu P_{i+1,1}+(i+1)\mu P_{i+1,0}+\beta P_{i,1}$$

$$\begin{aligned}(\lambda+i\mu+(i-j)\alpha+j\beta)P_{i,j} &= \lambda P_{i-1,j-1}+(i+1)\mu P_{i+1,j+1}+(j+1)\beta P_{i,j+1} \\ &\quad + (i+1-j)\alpha P_{i,j-1} \quad : \quad 1\leq j\leq i-1 \end{aligned} \tag{2}$$

$$(\lambda+i\mu+i\beta)P_{i,i} = \lambda P_{i-1,i-1}+(i+1)\mu P_{i+1,i+1}+\alpha P_{i,i-1} \cdot$$

For $C \le i \le N-1$ we have

$$(\lambda+i\mu+i\alpha)P_{i,0} = \beta P_{i,1}+(i+1)\mu P_{i+1,0}+(i+1)\mu P_{i+1,1}$$

$$(\lambda+i\mu+(i-j)\alpha+j\beta)P_{i,j} = \lambda P_{i-1,j-1}+(i+1)\mu P_{i+1,j+1} + (i+1-j)\alpha P_{i,j-1}+(j+1)\beta P_{i,j+1} \; : \; 1 \le j \le C-1 \tag{3}$$

$$(\lambda+i\mu+(i-j)\alpha+C\beta)P_{i,j} = \lambda P_{i-1,j-1}+(i+1)\mu P_{i+1,j+1}+C\beta P_{i,j+1} + (i+1-j)\alpha P_{i,j-1} \qquad C \le j \le i-1$$

$$(\lambda+i\mu+C\beta)P_{i,i} = \lambda P_{i-1,j-1}+(i+1)\mu P_{i+1,i+1}+\alpha P_{i,i-1};$$

and finally for i=N we have

$$(N\mu+N\alpha)P_{N,0} = \beta P_{N,1}$$

$$(N\mu+(N-j)\alpha+j\beta)P_{N,j} = \lambda P_{N-1,j-1}+(N+1-j)\alpha P_{N,j-1}+(j+1)\beta P_{N,j+1} \; : \; 1 \le j \le C-1$$

$$(N\mu+(N-j)\alpha+C\beta)P_{N,j} = \lambda P_{N-1,j-1}+(N+1-j)\alpha P_{N,j-1}+C\beta P_{N,j+1} \; : \; C \le j \le N-1 \tag{4}$$

$$(N\mu+C\beta)P_{N,N} = \lambda P_{N-1,N-1}+\alpha P_{N,N-1} \, .$$

Before proceeding let us consider two special cases; the first occurs when N=C=1. From equation (2) we have

$$\rho P_{0,0} = P_{1,0}+P_{1,1} = Pr\{Q=1\} = \frac{\rho}{1+\rho}$$

or

$$P_{0,0} = \frac{1}{1+\rho} \tag{5}$$

which is to be expected, since $P_{0,0} = Pr\{Q=0\}$. Now from equation (3) and from the fact that $Pr\{Q=1\}=\rho/(1+\rho)$, one gets

$$P_{1,0} = \frac{\rho\beta}{(1+\rho)(\mu+\alpha+\beta)} \tag{6}$$

and

$$P_{1,1} = \frac{\rho(\mu+\alpha)}{(1+\rho)(\mu+\alpha+\beta)} . \tag{7}$$

The second special case we consider occurs when α and β get large while $\eta=\alpha/\beta$ remains constant. Equations (2), (3), (4) become for $i \leq C-1$,

$$\begin{aligned}
\rho P_{0,0} &= P_{1,0}+P_{1,1} \\
i\eta P_{i,0} &= P_{i,1} \\
((i-j)\eta+j)P_{i,j} &= (i+1-j)\eta P_{i,j-1}+(j+1)P_{i,j+1} \quad : \quad 1 \leq j \leq i-1 \\
iP_{i,i} &= \eta P_{i,i-1}.
\end{aligned} \tag{8}$$

Again, for $C \leq i \leq N-1$ we have

$$\begin{aligned}
i\eta P_{i,0} &= P_{i,1} \\
((i-j)\eta+j)P_{i,j} &= (i+1-j)\eta P_{i,j-1}+(j+1)P_{i,j+1} \quad : \quad 1 \leq j \leq C-1 \\
((i-j)\eta+C)P_{i,j} &= (i+1-j)\eta P_{i,j-1}+CP_{i,j+1} \quad : \quad C \leq j \leq i-1 \\
CP_{i,i} &= \eta P_{i,j-1},
\end{aligned} \tag{9}$$

and finally

$$\begin{aligned}
N\eta P_{N,0} &= P_{N,1} \\
((N-j)\eta+j)P_{N,j} &= (N+1)\eta P_{N,j-1}+(j+1)P_{N,j+1} \quad : \quad 1 \leq j \leq C-1 \\
((k-j)\eta+C)P_{N,j} &= (N+1-j)\eta P_{N,j-1}+CP_{N,j+1} \\
CP_{N,N} &= \eta P_{N,N-1}.
\end{aligned} \tag{10}$$

We note that these equations are decoupled in i, and the solution is easily seen to be for $i \leq C$

$$P_{i,j} = \binom{i}{j} \eta^j P_{i,0} \tag{11}$$

and for $i \geq C$

$$P_{i,j} = \begin{cases} \binom{i}{j} \eta^j P_{i,0} & : j \leq C \\ \dfrac{i!}{(i-j)!\,C!\,C^{j-C}} \eta^j P_{i,0} & : j \geq C \end{cases} \tag{12}$$

where $P_{i,0}$ is found using $Pr\{Q=i\} = \sum_{j=0}^{i} P_{i,j}$ to be

$$P_{i,0} = \begin{cases} \dfrac{Pr\{Q=i\}}{(1+\eta)^i} & i \leq C \\ \\ \dfrac{Pr\{Q=i\}}{\sum_{r=0}^{C-1} \binom{i}{r} \eta^r + \sum_{r=C}^{i} \dfrac{i!\,\eta^r}{(i-r)!\,C!\,C^{r-C}}} & i \geq C \end{cases} \tag{13}$$

where $Pr\{Q=i\}$ is given by equation (1).

From equations (11)-(13), one sees that for this special case $P_{i,j}$ can be expressed as the product of the probability the number of voice calls in the system equals i ($Pr\{Q=i\}$), and the state probability of a finite

source, C-server queueing system [7].

Let us now consider the extent of the difference between the results one gets when α and β are large and the actual results one would get without taking this limit. For this comparison we use the first special case and consider the expected number of talkspurts in the system, $E\{Q_T\}$, from equation (7) we have

$$E\{Q_T\} = \frac{\rho}{1+\rho}\;\frac{\mu+\beta}{\mu+\alpha+\beta}\,. \tag{14}$$

But from specializing the results for the second special case we get

$$E\{Q_T\} = \frac{\rho}{1+\rho}\;\frac{\eta}{1+\eta}. \tag{15}$$

From [4], the nominal values of α^{-1} and β^{-1} are 1.34 and 1.23 seconds; if one assumes μ^{-1}=180 seconds, then $E\{Q_T\}$=.4805Pr{Q=1} and .4786Pr{Q=1} from equations (14) and (15) respectively. The difference between these two quantities is approximately .0019 Pr{Q=1} and so is very minimal. Thus, for the values of α, β, and μ given above, one would expect the actual solution to equations (2)-(4) to be very close to the solution given by equations (11)-(13) and we use those results for the remainder of the technical note.

Some expected value results are now easily written down. The expected number of talkspurts in the system, $E\{Q_T\}$, is

$$E\{Q_T\} = \sum_{i=0}^{N} \sum_{j=0}^{i} jP_{i,j}; \tag{16}$$

the expected number of talkspurts in queue, $E_q\{Q_T^q\}$, is

$$E_q\{Q_T^q\} = \sum_{i=C+1}^{N} \sum_{j=C}^{i} (j-C)P_{i,j}, \tag{17}$$

and the average waiting time of a talkspurt in the buffer, $E\{W_T^q\}$, is

$$E\{W_T^q\} = E_q\{Q_T^q\}/\overline{\alpha} \tag{18}$$

where

$$\overline{\alpha} = \alpha \sum_{i=0}^{N} \sum_{j=0}^{i} (i-j)P_{i,j} \tag{19}$$

and Q_T^q and W_T^q are the random variables representing the number of talkspurts in the buffer and the waiting time of a talkspurt in the buffer.

We now develop an equation for the probability the waiting time in the buffer is less than or equal to t, $\Pr\{W_T^q \leq t\}$. In order to do so, we need an expression for the probability Q=i, and an arriving talkspurt that is accepted into the system finds j talkspurts already there; let us denote this probability by $R_{i,j}$. Following Gross and Harris [8] we have

$$R_{i,j} = \frac{(i-j)P_{i,j}}{\sum_{i=0}^{N} \sum_{r=0}^{i} (i-r)P_{i,r}} \qquad \begin{array}{l} i=0,1,\ldots N \\ i=0,1,\ldots,i-1, \end{array} \tag{20}$$

and so

$$\Pr\{W_T^q \leq t\} = 1- \sum_{i=C+1}^{N} \sum_{j=C}^{i} R_{i,j} \sum_{r=0}^{j-C} e^{-\beta t} \frac{(c\beta t)^r}{r!} . \tag{21}$$

The probability of zero wait in the buffer is

$$\Pr\{W_T^q=0\} = 1-\sum_{i=C+1}^{N}\sum_{j=C}^{i} R_{i,j} \tag{22}$$

The probability distribution of the total waiting time in the system for a talkspurt is found by convoluting $\Pr\{W_T^q \leq t\}$ with the exponential service time of a talkspurt.

An interesting modification to this system would be one where the talkspurts are guaranteed not to wait longer than T_0 seconds in buffer on the average. The way to ensure this is to fix the maximum number of talkspurts allowed in the system at L where L is determined by the largest integer such that

$$\frac{(L-C)}{C\beta} \leq T_0 . \tag{23}$$

Specializing equations (8)-(10) to reflect this finite buffer,one gets for $i \leq C$

$$P_{i,j} = \binom{i}{j}\eta^j P_{i,0} , \tag{24}$$

and for $C+1 \leq i \leq N$

$$P_{i,j} = \begin{cases} \binom{i}{j}\eta^j P_{i,0} & j \leq C \\ \dfrac{i!}{(i-j)!\,C!\,C^{j-C}}\eta^j P_{i,0} & C \leq j \leq L \end{cases} \tag{25}$$

where

$$P_{i,0} = \begin{cases} \dfrac{\Pr\{Q=i\}}{(1+\eta)^i} & : \quad i \leq C \\ \dfrac{\Pr\{Q=i\}}{\sum_{r=0}^{C-1}\binom{i}{r}\eta^r + \sum_{r=C}^{\min(i,L)} \dfrac{i!\,\eta^r}{(i-r)!\,C!\,C^{r-C}}} & : \quad C \leq i \end{cases} \tag{26}$$

The measures of performance are found as in the previous case except that the upper limit of the second summation in equation (17) should be min(i,L) instead of i, and the upper limit, i, of the summation in equations (19), (20), (21), and (22), should be replaced by min(i,L)-1.

For this modification another measure of performance that is very important is the portion of talkspurts that overflow or are frozen out of the system by the finite buffer, P_{ovf}. Using the carried load concept, [6] to determine P_{ovf}, we have

$$P_{ovf} = 1 - \frac{\text{Expected no. of busy servers}}{\text{Expected offered load}}$$

$$= 1 - \frac{\sum_{i=0}^{N} \sum_{j=0}^{\min(i,L)} \min(j,C) P_{i,j}}{\eta \sum_{i=0}^{K} \sum_{j=0}^{\min(i,L)} (i-j) P_{i,j}} . \quad (27)$$

In the next section, we use these results on several numerical problems of interest. In this section, we also relate the results we have obtained with the results that have been previously obtained for the packet switching of the talkspurt.

III. SYSTEM ANALYSIS

In the previous section, we developed some simple equations for various measures of performance for the behavior of a talkspurt in a system where the voice calls are coming and going via a statistical process. As pointed out in section I, the switching technology that would allow one to buffer talkspurts is the packetization of the active portion of a voice conversation. The analysis of a packet voice system has been accomplished in [4] and [5] when the number of voice calls present in the system was held fixed. We were interested in considering the same system where the presence of the voice calls may statistically vary. This required that we consider the performance of the system at the talkspurt level. If we extended the analysis down to the packet level, one more variable would have to be added to $P_{i,j}$ and the resultant required analysis would be increased in complexity. There are some general conclusions one can make regarding the result we obtained and those obtained in [4] and [5]. As the following curves are presented these comments are made.

Figure 1 gives a comparison of the results we obtained with our model (the dashed lines) and the results that Lincoln Labs obtained when they considered packet-switching of talkspurts. These results were taken from their report [4] and were obtained via simulation. The average delay in the buffer in milliseconds is plotted versus the TASI advantage (N/C) for different values of C. In order to reduce our model to theirs, we had to allow the voice load, ρ, to be large for a particular value of N and C. As one can see, the average delay for the talkspurt model is always lower than their results, and closely agree with their results when the average delay is less than 50 msec. One

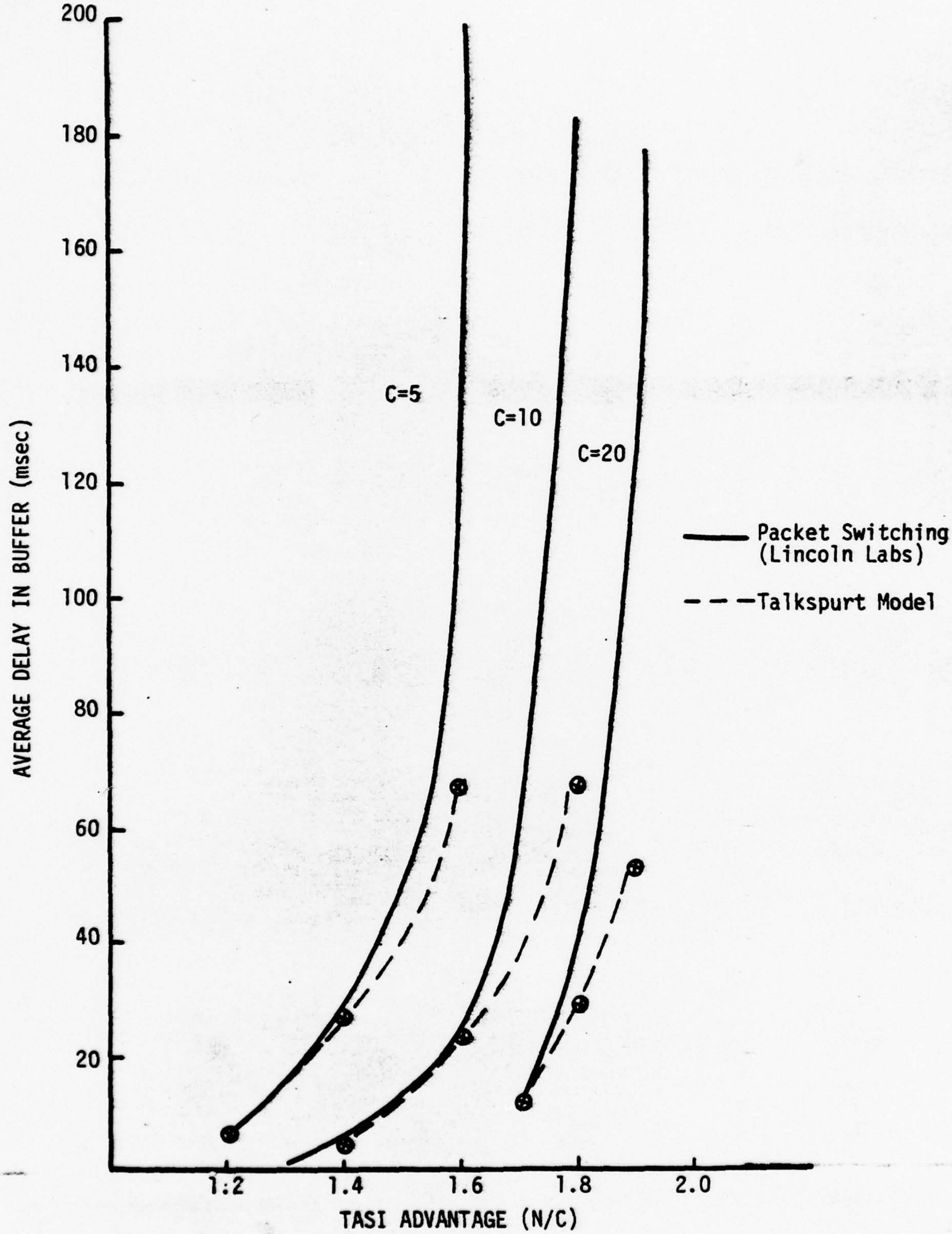

Figure 1. Comparison of Expected Waiting Time Results via Packet-Switching and Talkspurt Model (α^{-1}=1.34 sec., β^{-1}=1.24 sec.)

reason the talkspurt model gives lower results is that the talkspurt model does not allow a conversation to generate additional talkspurts when it is in the talkspurt mode. The packet-switch model does not impose this constraint, and hence, allows for large buffer buildup. This fact was also pointed out in reference [4].

Figure 2 gives a comparison similar to that made in figure 1 for the probability of overflow. For various values of TASI advantage, the probability of overflow is given as a function of maximum buffer delay, T_0. For the 100 and 500 msec. cases, the results for the packet-switched model are given in the parenthesis. That is, for a TASI advantage of 1.375 and T_0=100 msec, the talkspurt model gets P_{ovf}=.013, whereas the packet-switched model is significantly lower (.005). As T_0 gets larger (see T_0=500 msec), the two model have much closer agreement, but with the talkspurt model yielding higher results. One possible explanation for this fact is that we discard the complete talkspurt and all the packets associated with it; whereas, in the packet model only those packets for which no buffers are available are discarded.

Figure 3 presents two results in terms of variability of the presence of voice calls. Three curves are presented for different values of voice blocking probability; for each curve the average delay in the buffer is given as a function of the TASI advantage. The values of PB are found via Erlang's Loss Formula, $E_B(\rho,N)$, and represent the portion of time all N calls are in the system. That is, for N/C=1.5, we have N=15 and PB=.2, implying that 20% of the time we have 15 calls present. Whereas for PB=1,

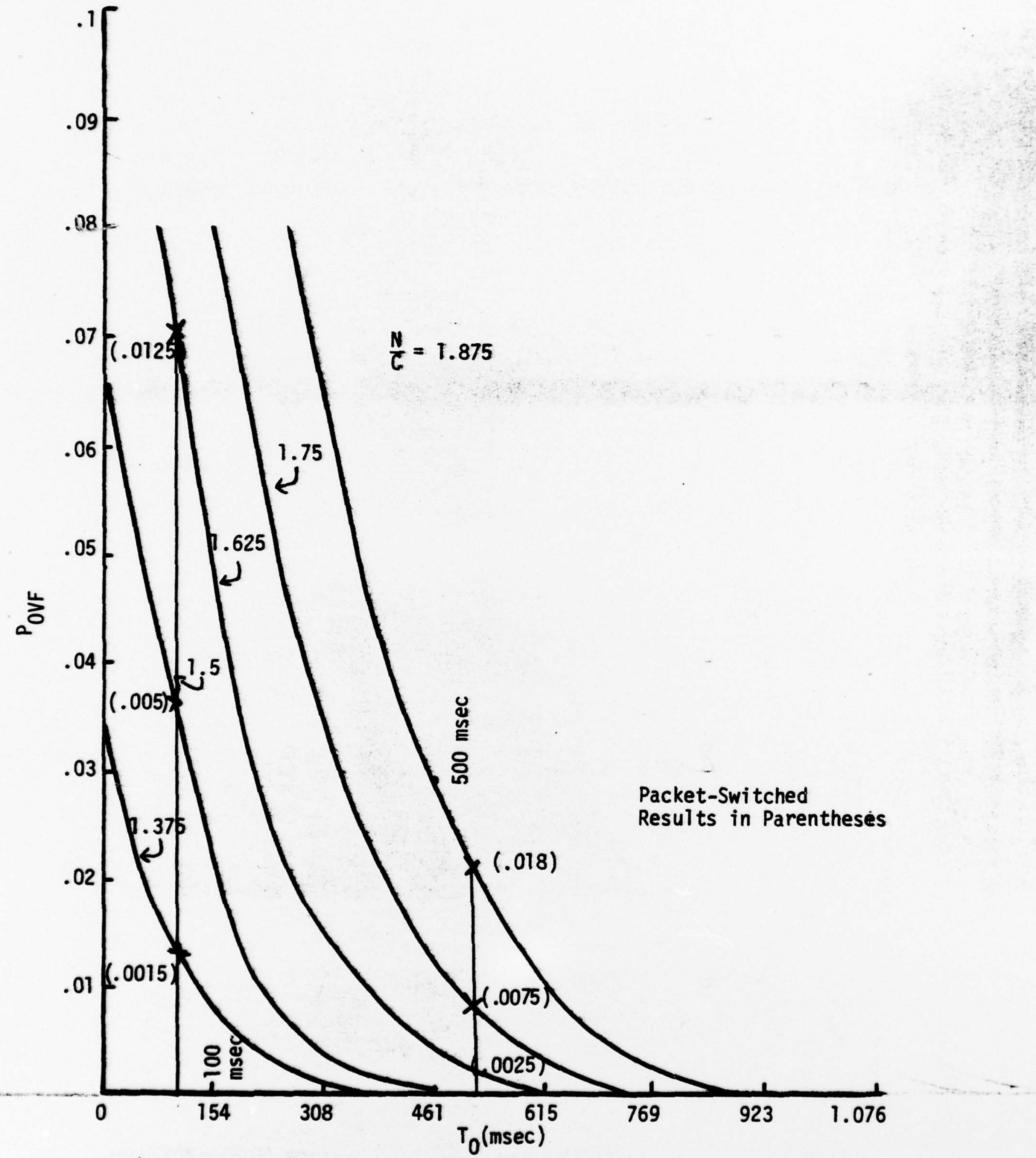

Figure 2. Comparison of Probability of Overflow Results via Packet-Switch and Talkspurt Model (α^{-1}=1.34 sec., β^{-1}=1.24 sec., C=8)

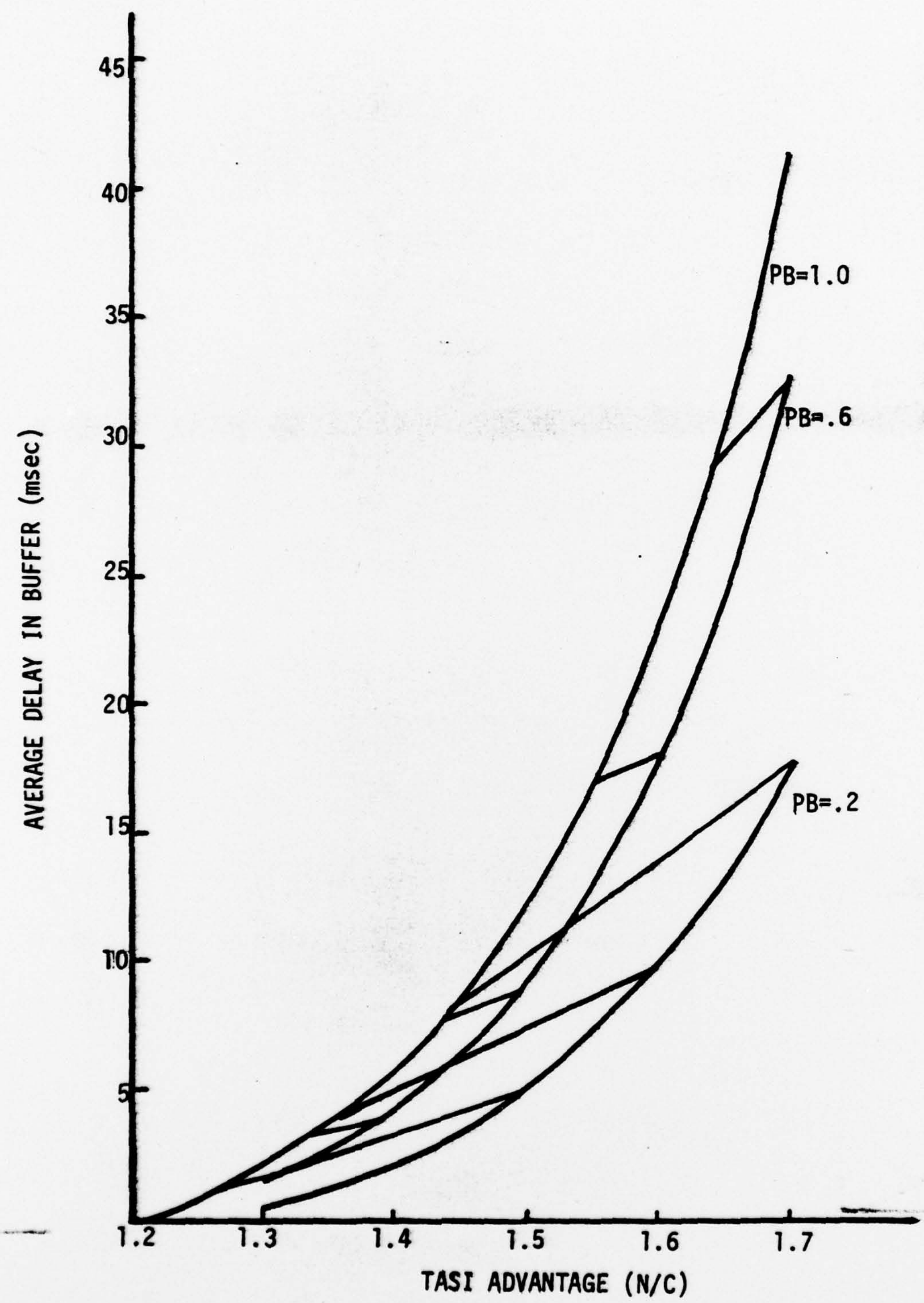

Figure 3. Sensitivity of Average Buffer Delay to Voice Blocking Probability (α^{-1}=1.34 sec., β^{-1}=1.23 sec., C=10)

100% of the time all 15 calls are present, i.e., the system considered in [1], [4], and [5]. As one would expect, the lower the value of PB the lower the expected delay in the buffer. Furthermore, the differences one gets in these delays increase significantly as the TASI advantage is increased.

Another point can be made from figure 3 by means of the lines connecting points on the PB=.6 and PB=.2 curves to points on the PB=1 curve. Consider the case of PB=.6 and N/C=1.7 or N=17; if ρ=40.97, then EB(40.97,17)=.6 and the expected number of calls in the system is 40.97(1-.6)=16.39. The expected delay in the buffer for the PB=.6 and N/C=1.7 case is compared, via the line connecting the two points, with the case where the number of talkers is fixed at 16.39; that is, N/C=1.639 on the PB=1 curve. From this comparison, one can see that the results one would get with the number of calls fixed is lower than those when the number of voice calls may vary but the expected number of calls which is present is the same as in the fixed case.

Figure 4 shows a tradeoff between maximum allowable delay in buffer and the probability of overflow for various values of voice blocking probability. This curve allows one to see if a tolerable probability of overflow and average delay in buffer can be obtained as a function of maximum buffer delay and voice blocking probability. For example, suppose one desires a probability of overflow less than or equal to .02; then the maximum buffer delay one can tolerate is around 205 msec with guaranteed average buffer delay of 6 msec or less for a voice blocking probability of .2. Figure 4 also shows that for maximum buffer delay of 500 msec or more, the expected buffer delay

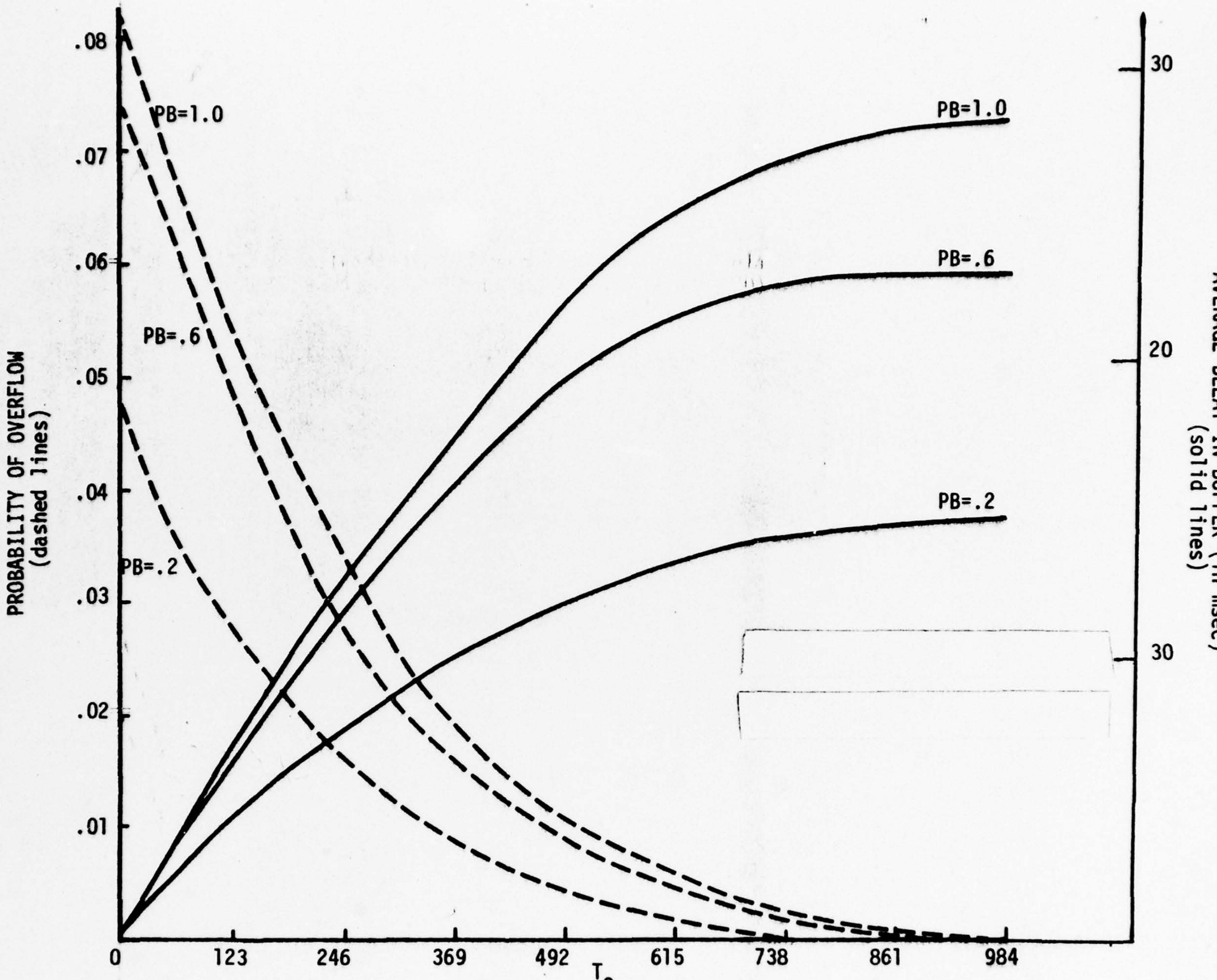

Figure 4. System Trade-Offs Between Maximum Buffer Delay and Probability of Overflow (α^{-1}=1.34 sec., β^{-1}=1.23 sec., N=36 and C=20)

and probability of overflow is relatively insensitive to changes in T_0. Whereas, for T_0 less than 500 msec, the sensitivity is significant and tradeoff has to be considered.

Figure 5 shows a family of curves that to the best of our knowledge has not been presented before in the literature for this type of system. Most engineering studies consider only the average delay or the variance of the delay; we give results for the probability distribution function of delay for various values of voice blocking. We note that it is the distribution function of the delay in the buffer and not of the whole system. The reason is, the behavior in terms of delay for the packet switch model and the talkspurt model should be similar when one only considers delay in the buffer. This is especially true for low average delays as pointed out earlier. If one wants to use our results to determine the delay distribution of the total waiting time of a packet in the system, denoted by $Pr\{W \le t\}$, then one would use the following formula:

$$Pr\{W \le t\} = \begin{cases} 0 & t<d \\ Pr\{W_T^q \le t-d\} & t>d \end{cases} \tag{28}$$

where d is the length of time to transmit a packet (assumed to be constant).

From the curves of figure 5 one can determine the time, $t_{.95}$, such that $Pr\{W_T^q \le t_{.95}\}=.95$. The lines at the top of the curves compare $t_{.95}$ with expected waiting time in the buffer for each of the three voice blocking probabilities. From this figure, $t_{.95}$ is around seven or eight times the expected delay in the buffer.

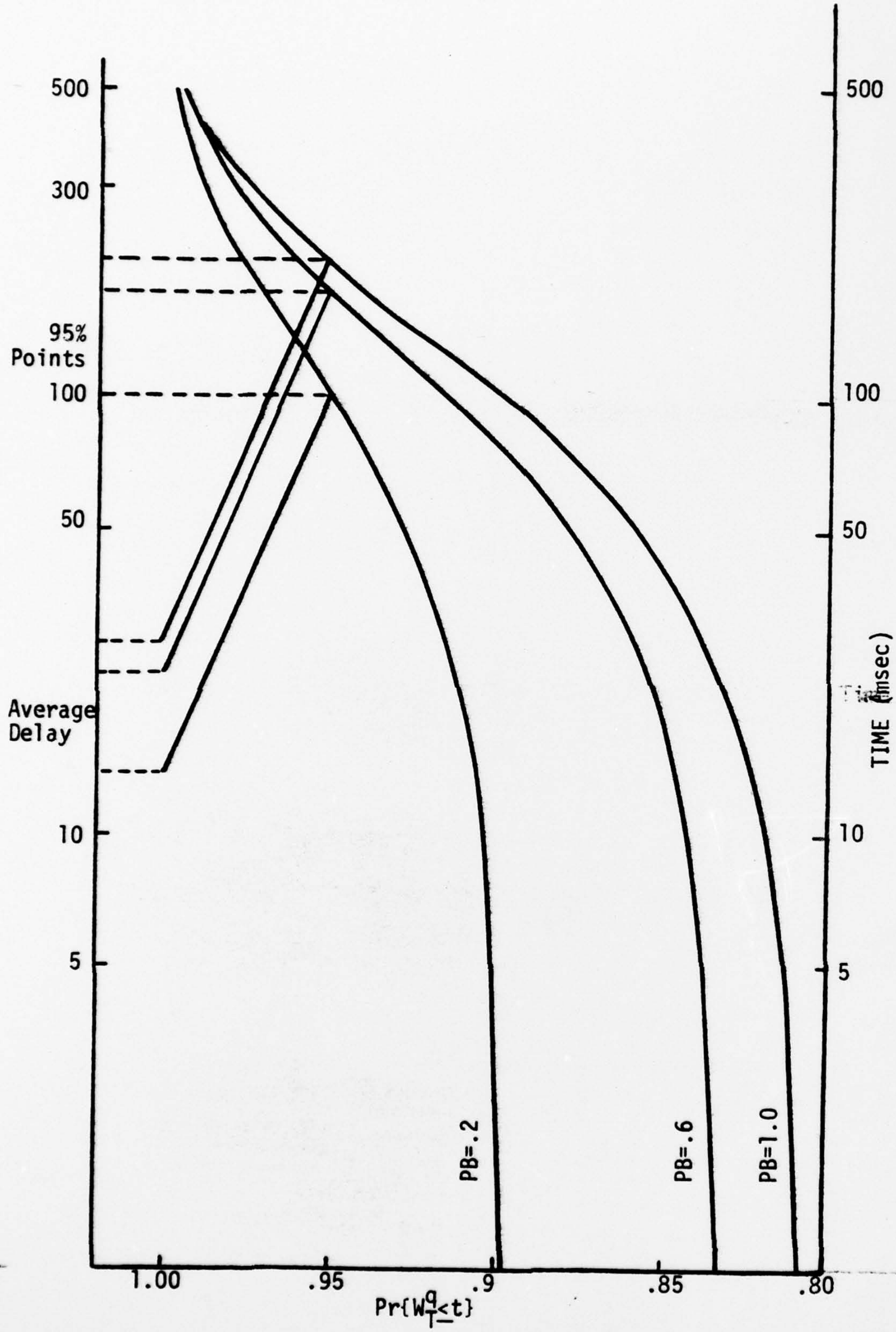

Figure 5. Cummulative Distribution Function of Waiting Time in Buffer (α^{-1}=1.34 sec., β^{-1}=1.23, N=36, C=20)

These results are further examined in figures 6 and 7. There we consider the sensitivity of $Pr\{W_T^q \leq t\}$ to small, C=5, and large, C=30, numbers of voice channels. As one can see, the sensitivity is much greater for the small number of voice channels than for the large number of channels. Several points can also be made from these figures; first, as the TASI advantage increases, the probability distribution, $Pr\{W_T^q \leq t\}$, decreases for a fixed value of t. Second, for the cases where it applies, the 95th percentile point is again around seven or eight times the expected delay in the buffer. Finally, these figures give us further insight into the findings of reference [4] which stated that the TASI advantage could be improved on even for a small number of voice channels if one would accept some delay. The dashed lines connect the expected delay in the buffer to the value of $Pr\{W_T^q \leq t\}$ evaluated at $t=E\{W_T^q\}$. That is, the percentage of the talkspurts (and/or packets since the two see the same delay) that see a delay is less than or equal to the mean. Even though this percentage varies with the TASI advantage, the more significant fact to be seen is that for the small number of voice channels this percentage is rather low and might not be acceptable in practice. Whereas, for a large number of voice channels this percentage is much higher and probably acceptable in practice. The point to be made is that even though improvements in the TASI advantage can be obtained for small numbers of voice channels, when one considers expected delay in the buffer, the percentage of packets that experience a delay larger than the average might be unacceptable.

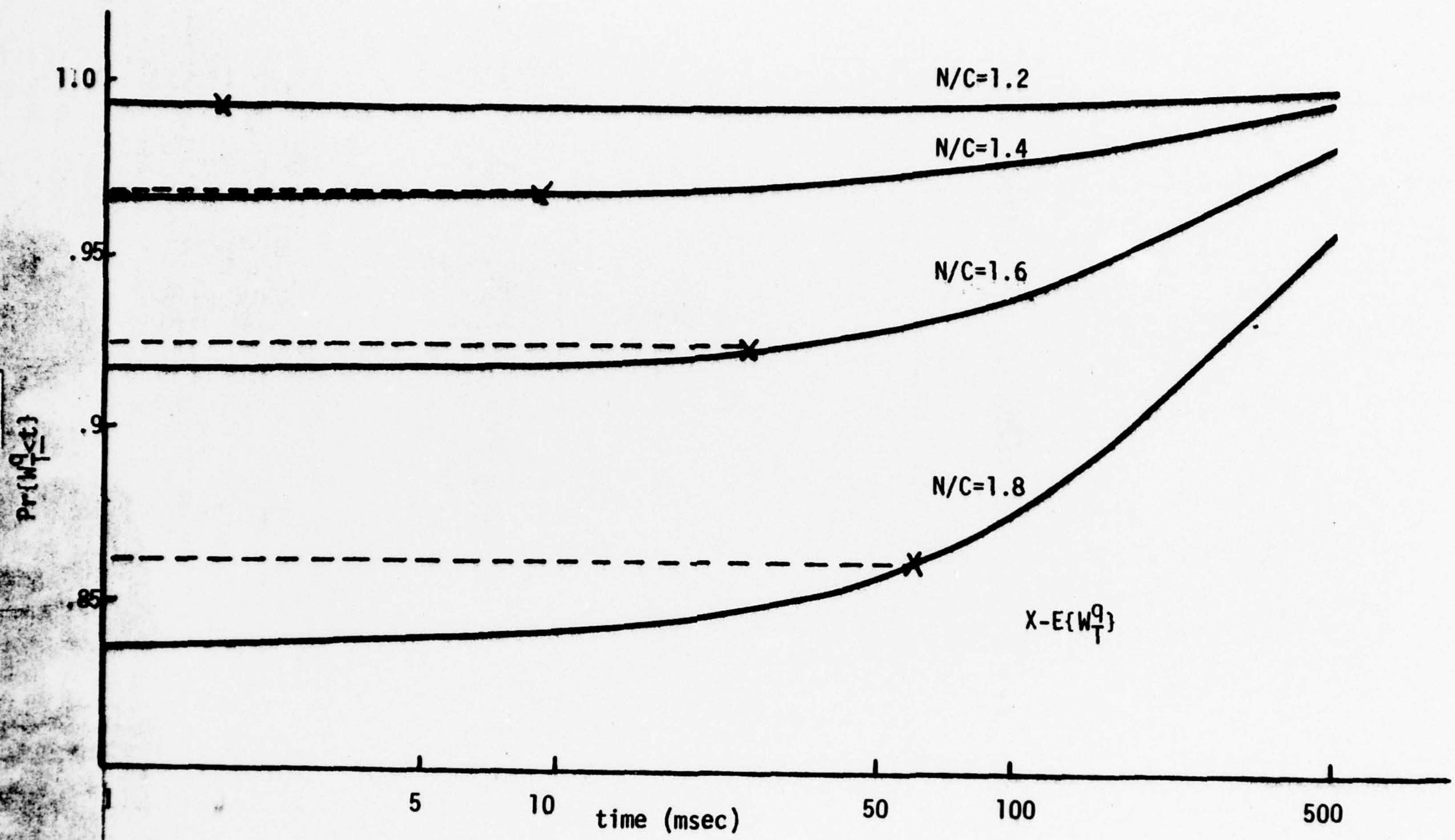

Figure 6. Sensitivity of Waiting Time Distribution for Small Number of Voice Channels (C=5)
(α^{-1}=1.34 sec., β^{-1}=1.23 sec., PB=.2)

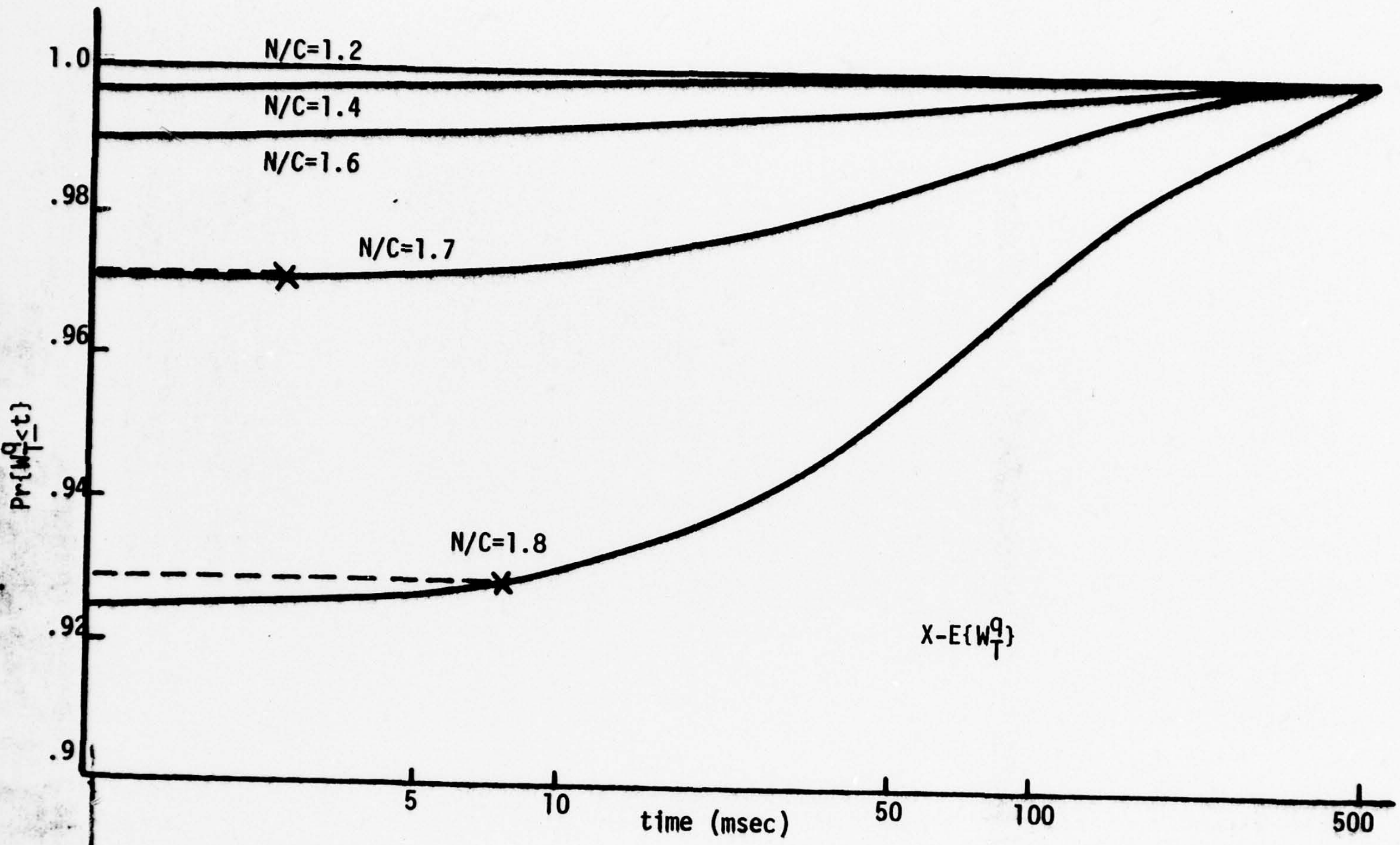

Figure 7. Sensitivity of Waiting Time Distribution for Large Number of Voice Channels (C=30) (α^{-1}=1.34 sec., β^{-1}=1.23, PB=.2)

IV. CONCLUSIONS

We have presented a mathematical model that predicts the performance of a talkspurt in a packetized voice communications system where voice calls are randomly allowed to come and go. Numerical applications of this model involved comparisons with previously generated results as well as new results for the complete probability distribution of delay. These results can be summarized as follows:

1. For average delays less than 50 msec, the talkspurt model and packet model give similar results (figure 1).

2. Significant differences in the average delay in the buffer can be obtained, depending on the blocking probability of voice calls (figure 3).

3. Fixing the number of voice calls in the system results in shorter average buffer delays than in a system where the calls randomly come and go but the expected number of calls present is the same as in the fixed system (figure 3).

4. Engineering tradeoff studies between maximum buffer delay, average buffer delay, and probability of overflow are easily made (figure 4).

5. New results for the probability distribution of delay in the buffer are easily computed and tabulated so that comparisons of expected delay and percentile points of this distribution can be made. For the examples considered here, the 95th percentile point was around seven times the expected delay (figure 5).

6. Previous analyses ([4] and [5]) for fixed numbers of calls have shown that substantial increases in the TASI advantage can be gained when the contents of talkspurts are buffered, even for a small number of voice

channels. Our analysis has shown that one gets shorter average delays and smaller overflow probabilities when the calls are allowed to randomly arrive and leave, thus, further strengthening these results. However, our new results indicate that the 95th percentile point of the waiting distribution is seven times the mean. Furthermore, the percentage of packets that see a greater than the average delay for the smaller number of voice channels might not be acceptable in practice (figures 6 and 7), thus, indicating that further analysis and study is required on this aspect of the problem.

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